

Analysis of the DENZ04 low-energy $\pi^\pm p$ elastic-scattering data

E. Matsinos^{*a}, G. Rasche^b,

^a*Centre for Applied Mathematics and Physics, Zurich University of Applied Sciences, Technikumstrasse 9, P.O. Box, CH-8401 Winterthur, Switzerland*

^b*Institut für Theoretische Physik der Universität, Winterthurerstrasse 190, CH-8057 Zürich, Switzerland*

^{*}Corresponding author. E-mail: evangelos.matsinos@zhaw.ch, evangelos.matsinos@sunrise.ch; Tel.: +41 58 9347882; Fax: +41 58 9357306

Abstract

This paper presents the results of an analysis of the DENZ04 [1] low-energy $\pi^\pm p$ differential cross sections. We first analysed separately the π^+p and the π^-p elastic-scattering measurements on the basis of standard low-energy expansions of the s - and p -wave K -matrix elements. After the removal of the outliers (eleven degrees of freedom in the initial database), we subjected the truncated $\pi^\pm p$ elastic-scattering databases into a common optimisation scheme using the ETH model [4]; the optimisation failed to produce reasonable values for the model parameters. The phase-shift solution, extracted from the model fit to the data, is very odd. The problems we have encountered in the analysis of the DENZ04 data are due to the shape of the angular distributions of their $\pi^\pm p$ differential cross sections.

PACS: 13.75.Gx; 25.80.Dj

Key words: πN elastic scattering

1 Introduction

This is the second of three papers addressing issues of the pion-nucleon (πN) interaction at low energies (pion laboratory kinetic energy $T \leq 100$ MeV). The goal in this study is to investigate the self-consistency of the $\pi^\pm p$ elastic-scattering differential cross sections (DCSs) of Ref. [1] (hereafter, referred to as DENZ04), which have not been included in our last two phase-shift analyses (PSAs) [2] and [3] (to be referred to as UZH06 and ZUAS12, respectively).

In the case of UZH06, we did not notice that the measurements were already available for some time ¹. In the case of ZUAS12, we made a conscious decision to avoid modifying our UZH06 database prior to the assessment of the self-consistency of any candidate additions, especially given the amount of the DENZ04 measurements. In the present work, we will give arguments supporting our position not to use the DENZ04 data and to retain our initial UZH06 $\pi^\pm p$ elastic-scattering databases.

We will analyse the DENZ04 data as if it comprised the entire $\pi^\pm p$ elastic-scattering database at low energies; this way, a possible failure when testing the integrity of these measurements cannot be blamed on other experimental data. We will follow the method which was introduced in Section 4 of Ref. [2] and developed to its current form in Ref. [3] (see Section 2 therein). We will first investigate the self-consistency of the π^+p measurements on the basis of suitable low-energy expansions of the s - and p -wave K -matrix elements; the most recent values of any constants, which are used in the parameterisation of these quantities, may be found in Ref. [3]. Any outliers will be removed from the data and the fits will be repeated until the data sets have been ‘cleaned up’ and are thus ready for submission to further analysis. At the next step, the π^-p elastic-scattering measurements will be analysed. After removing any outliers also from these data, we will investigate the possibility of analysing both reactions in a common optimisation scheme; in that part of the study, we will use both the low-energy expansions of the s - and p -wave K -matrix elements and the ETH model [4].

The last part of the study will be dedicated to the reproduction of the absolute normalisation of the DENZ04 data on the basis of our ZUAS12 solution. We will show that the shape of the angular distributions of most of the DENZ04 π^+p data sets is not compatible with the bulk of our established π^+p database at low energies (which led to the UZH06 and ZUAS12 solutions).

2 Method

The determination of the observables from the hadronic phase shifts has been given in detail in Section 2 of Ref. [2]. For π^+p scattering, one obtains the partial-wave amplitudes from Eq. (1) of that paper and determines the no-spin-flip and spin-flip amplitudes via Eqs. (2) and (3). Finally, the observables are evaluated from these amplitudes via Eqs. (13) and (14). For π^-p elastic scattering, the observables are determined on the basis of Eqs. (15-20).

¹ The 546 DENZ04 DCS values have been given in tabulated form in Denz’s dissertation, and were available two years prior to the main publication of the CHAOS Collaboration.

All the details on the analysis method (i.e., on the minimisation function, on the definitions of the scale factors, etc.) may be found in Section 2.2 of Ref. [3]. The contribution χ_j^2 of the j^{th} data set to the overall χ^2 is given therein by Eq. (1). The scale factors z_j , which minimise each χ_j^2 , are evaluated using Eq. (2); the minimal χ_j^2 value for each data set (denoted by $(\chi_j^2)_{min}$) is given in Eq. (3) and the scaling contribution (of the j^{th} data set) to $(\chi_j^2)_{min}$ in Eq. (4). Finally, the scale factors for free floating \hat{z}_j (which we will use in Section 3.4.2, when investigating the absolute normalisation of the DENZ04 data sets using the ZUAS12 solution as reference) are obtained via Eq. (5); their total uncertainty $\Delta\hat{z}_j$ has been defined at the end of Section 2.2 of Ref. [3].

One statistical test will be performed for each data set, the one involving its contribution $(\chi_j^2)_{min}$ to the overall χ^2 . The corresponding p-value will be evaluated on the basis of $(\chi_j^2)_{min}$ and of the number of degrees of freedom of the data set (hereafter, the acronym DOF will stand for ‘degree(s) of freedom’, whereas NDF will denote the ‘number of DOF’); for a data set with N_j data points (none of which is an outlier), NDF is equal to N_j . The p-value for each data set will be compared to the confidence level p_{min} for the acceptance of the null hypothesis (implying no statistically-significant effects). The value of p_{min} is fixed to the equivalent of a 2.5σ effect in the normal distribution, corresponding to about $1.24 \cdot 10^{-2}$.

The repetitive use of the full description of the databases is largely facilitated if we adhere to the following notation: DB_+ for the π^+p database; DB_- for the π^-p elastic-scattering database; $DB_{+/-}$ for the combined $\pi^\pm p$ elastic-scattering databases.

3 The DENZ04 data

The DENZ04 DB_+ consists of 275 data points, acquired at five energies between 19.90 and 43.30 MeV. Two sets of values are available at 43.30 MeV: one was taken at the same conditions as the data at the lower energies, whereas another was obtained with the target rotated by 64° ; similarly to the notation in Denz’s dissertation, we will identify the data set corresponding to the rotated target via the label ‘(rot.)’. Technically, the DENZ04 π^+p data must be assigned to only 6 data sets [1]. However, the measurements have been assigned to a total of 17 data sets in the SAID database [5], after splitting each data set (except at 19.90 MeV) into three parts: forward-angle ($\theta < 35^\circ$), medium-angle ($35 < \theta \lesssim 150^\circ$), and backward-angle ($\theta \gtrsim 150^\circ$); θ denotes the centre-of-mass (CM) scattering angle. The 19.90 MeV data do not cover scattering angles above 98.45° ; therefore, the original data set has been split into two segments. Although we have analysed the DENZ04 data also the way the CHAOS Collaboration appears to recommend (i.e., by assigning the

measurements for each reaction to only 6 data sets), the results of our analysis clearly favour the splitting of the data sets into segments the way this is done in the SAID database. We therefore decided to present results following the SAID assignment of the DENZ04 π^+p measurements to 17 data sets. The assignment of the measurements to 17 data sets enables the determination of the scale factors z_j from data which are more ‘localised’ (in terms of the scattering angle) and, as such, it implies a more benign treatment of the data. Evidently, the appearance of any problems in case of the assignment of the data to 17 data sets can only be exacerbated if only 6 data sets are used.

The DENZ04 DB₋ comprises 271 data points, taken at the same five beam energies of the DENZ04 DB₊; similarly to π^+p , two sets of DCS values have been acquired at 43.30 MeV. In SAID, the measurements have been assigned to 12 data sets; we will do the same.

The final normalisation uncertainties reported by the CHAOS Collaboration [1] are as follows: 5% at the three lowest energies and 7% for the 43.30 MeV data sets. Asymmetric uncertainties have been given for the 37.10 MeV data sets (+5, −9%); unable to treat asymmetric uncertainties in our software, we have decided to use the normalisation uncertainty of 7% (average of the two absolute values) for the 37.10 MeV data.

We found that the current SAID solution (WI08) has been obtained with the *wrong* assignment of the normalisation uncertainties of the DENZ04 data ². Given the largeness of the DENZ04 data set (it is almost equal in size to the rest of the DB_{+/-} for $T \leq 100$ MeV), the impact of this erroneous assignment on the WI08 solution at low energies can only be assessed by the proper re-analysis of the data.

The SAID output for the DENZ04 π^+p data also contains the contribution of these data sets to the overall χ^2 value. According to the results, the contribution of the 274 data points ³ of the DENZ04 π^+p data to the overall χ^2 is enormous (664.34 units). Furthermore, the χ^2 value for the DENZ04 π^-p elastic-scattering data is not much better (479.34 for 271 data points). On the basis of these numbers, it is evident that the WI08 phase-shift solution yields a very poor reproduction of the DENZ04 data.

² The SAID group have been using the normalisation uncertainties appearing in the captions of the tables of Appendix B of Denz’s dissertation (where the numerical values of the DCS measurements may be found), yet they have swapped the values which are applicable to the data for $\theta < 35^\circ$ and $\theta > 35^\circ$.

³ The SAID group have excluded one data point belonging to the 37.10 MeV medium-angle data set.

3.1 Fits to the DENZ04 DB₊ using the K -matrix expansions

The parameterisation of the s - and p -wave K -matrix elements for the low-energy π^+p scattering may be found in Section 3.1 of Ref. [3]. The optimal values of the corresponding seven parameters ($\tilde{a}_{0+}^{3/2}$, b_3 , c_3 , d_{33} , e_{33} , d_{31} , and e_{31}) are obtained via the minimisation of the χ^2 function (see Section 2.2 of Ref. [3]). We will apply the same acceptance criteria to the DENZ04 measurements which were applied to the data in the ZUAS12 PSA.

The results of the optimisation procedure are shown in Table 1. Since seven parameters are used to generate the fitted values, the NDF in the first fit to the DENZ04 DB₊ was 268; the minimum value of χ^2 was 486.9. For the truncated DENZ04 DB₊, the minimum value of χ^2 was 401.2 for 260 DOF in the fit. We noticed that the values of the seven parameters of the fit came out different from those obtained in the fits to the truncated DB₊ of Ref. [3]. The details on the truncated DENZ04 DB₊, as obtained from the final fit, are given in Table 2.

3.2 Fits to the DENZ04 DB₋ using the K -matrix expansions

The $I = 3/2$ amplitudes were fixed from the final fit to the truncated DENZ04 DB₊ and were imported into the analysis of the DENZ04 DB₋. The parameterisation of the s - and p -wave $I = 1/2$ K -matrix elements, suitable for the low-energy π^-p elastic scattering, may be found in Section 3.2 of Ref. [3]. Seven new parameters ($\tilde{a}_{0+}^{1/2}$, b_1 , c_1 , d_{13} , e_{13} , d_{11} , and e_{11}) are introduced at this stage. We present the steps in the process of removing outliers from the DENZ04 DB₋ in Table 3; only three data points had to be removed. We noticed that the final result for \tilde{a}^{cc} , obtained from the data, was very odd, around $0.041 \mu_c^{-1}$ (μ_c denotes the mass of the charged pion); the \tilde{a}^{cc} result of Ref. [3] is $0.0803(11) \mu_c^{-1}$. We traced this difference to the very unusual final values of the parameters entering the modelling of the s - and p -wave K -matrix elements for the π^+p reaction (i.e., the values which fixed the $I = 3/2$ amplitudes in the case of the fits to the DENZ04 DB₊). The details on each data set of the truncated DENZ04 DB₋, as obtained from the final fit, are given in Table 4.

3.3 Common fit to the DENZ04 DB_{+/-} using the *K*-matrix expansions

In order to give the two elastic-scattering reactions equal weight, we multiplied $(\chi_j^2)_{min}$ for each π^+p data set by

$$w_+ = \frac{N_+ + N_-}{2N_+}$$

and for each π^-p elastic-scattering data set by

$$w_- = \frac{N_+ + N_-}{2N_-},$$

where N_+ and N_- represent the NDF in the two databases; we then added these quantities for all the data sets to obtain the overall χ^2 value.

The common fit to the truncated DENZ04 DB_{+/-} was subsequently performed, using 14 parameters. This step was taken in order to examine whether any additional points (or data sets) had to be removed; none were identified. The common fit to the data yielded a χ^2 value of 751.5 for 521 DOF in the fit.

3.4 Common fit to the truncated DENZ04 DB_{+/-} using the ETH model

So far in this paper, we have used standard low-energy expansions of the πN amplitudes in terms of the pion CM kinetic energy. We will now use the ETH model which is based on Feynman diagrams. Details on the model, as well as on its seven parameters (G_σ , K_σ , G_ρ , K_ρ , $g_{\pi NN}$, $g_{\pi N\Delta}$, and Z) may be obtained from Refs. [2,3]. This model was introduced in Ref. [4] and was developed to its final form by the mid 1990s.

3.4.1 Results

The common fit of the ETH model to the truncated DENZ04 DB_{+/-} yielded a χ^2 value of 783.3 for 528 DOF in the fit. We discovered that there had been enormous problems during the fit. All results for the parameters of the ETH model turned out to be far from their ‘established’ values. Furthermore, the positivity of the correlation matrix was automatically enforced in the MINUIT library; this is evidence that the optimisation algorithms were trapped in a spurious χ^2 minimum.

Our results for the seven model parameters have shown remarkable stability over the years, from the period when the fits were performed to old, outdated phase shifts to the present times when the fits are made directly to the contents of the low-energy πN database. The database itself has also changed significantly over the last two decades. In any case, it is fair to say that, no matter which data were fitted to, the results for the model parameters always came out ‘reasonable’; this statement is valid even when obvious outliers (e.g., the measurements of Ref. [6]) were included in our database (e.g., see Ref. [7]).

The results for the model parameters, obtained from the common fit to the truncated DENZ04 DB_{+/-}, are very odd. Evidently, for whichever reasons, the parameters drift away from their ‘established’ values, to unreasonable (or even unphysical) ones. The outcome of the optimisation looks as if something has gone wrong during the fitting process. We therefore refrain from giving the ‘optimal’ values for the model parameters.

Despite the drift of the model parameters in the fit, we nevertheless decided to determine the s - and p -wave phase shifts with the model-parameter values obtained in the fit to the truncated DENZ04 DB_{+/-}. (Given that the evaluation of the correlation matrix had failed, we could not obtain reliable uncertainties.) The final results for these phase shifts were so different from the values extracted in Refs. [3,5] that we refrain from listing or plotting them. The values of the DENZ04-based hadronic phase shifts (and their overall tendency with increasing energy) were found very puzzling.

3.4.2 *Reproduction of the DENZ04 data on the basis of the ZUAS12 solution*

Given all these problems, we decided to investigate the reproduction of the DENZ04 data on the basis of the ZUAS12 phase-shift solution. Our goal in this part of the study is to identify the kinematical region(s) in which the DENZ04 data are poorly reproduced; if successful, we could pinpoint the origin of the problems we have encountered in the analysis of these measurements.

We will start with the reproduction of the measurements when the data sets are characterised only by the target configuration and the energy of the incident beam (i.e., following the recommendation of the CHAOS Collaboration [1]). The results are shown in Table 5. We notice that the overall χ^2 values of the reproduction (i.e., the sums of the corresponding $(\chi_j^2)_{min}$ values given in the table for the two elastic-scattering reactions) are: 547.7 for π^-p elastic scattering and (an astonishing) 2446.0 for π^+p .

The reproduction of the DENZ04 measurements after the data sets have been split into 29 segments in total (see Section 3) are given in Table 6. We observe that the overall χ^2 values of the reproduction drop for both reactions: to 506.5 for π^-p elastic scattering, to 747.3 for π^+p . The dramatic decrease in the latter

case is indisputable evidence that the problems we have encountered in the analysis of the DENZ04 data are mainly due to the shape of the π^+p angular distributions. The decrease in the case of the π^-p elastic-scattering data (i.e., unsplit versus split data sets) is very moderate, indicating considerably fewer problems with the DENZ04 DB₋. The results after removing the outliers, detailed in Tables 1 and 3, are shown in Table 7; the χ^2 values drop to 469.1 and 665.7 for π^-p and π^+p elastic scattering, respectively. The scale factors for free floating \hat{z}_j , corresponding to the optimal reproduction of the absolute normalisation of the DENZ04 data on the basis of the ZUAS12 solution are given in Figs. 1 and 2, separately for the two elastic-scattering reactions. For π^+p scattering, three \hat{z}_j values per energy are obtained (corresponding to the three angular intervals into which the measurements have been split, i.e., forward, medium, and backward angles); the 19.90 MeV data set does not cover backward angles. For π^-p elastic scattering, two \hat{z}_j values per energy are obtained (corresponding to the two angular intervals of the measurements, i.e., forward and medium/backward angles).

We recollect that the χ^2 results of Ref. [3] (for $p_{min} \approx 1.24 \cdot 10^{-2}$) for the two reactions were: 371.0 and 427.2 for 321 and 333 DOF in the fit, for π^-p and π^+p elastic scattering, respectively. The F -test performed on the two π^-p elastic-scattering databases (i.e., on the truncated DENZ04 DB₋ and on the truncated ZUAS12 DB₋) results in the score value of 1.515 for 268 and 321 DOF, corresponding to the p-value of about $1.9 \cdot 10^{-4}$. On the other hand, the F -test performed on the two corresponding π^+p databases results in the score value of 1.944 for 267 and 333 DOF, corresponding to the p-value of about $4.8 \cdot 10^{-9}$. These two results are sufficient to substantiate our position that the DENZ04 measurements are not compatible with the rest of the low-energy πN database, as it emerged in our PSA of Ref. [3]. In view of these striking differences, it makes no sense to include even part of the DENZ04 data, as they currently stand, in our PSAs.

The inspection of Fig. 1 shows that the extracted scale factors \hat{z}_j scatter wildly, almost as if they had been obtained via a stochastic process. A closer look, however, at the five entries corresponding to backward scattering demonstrates that these data can be reproduced well by our ZUAS12 solution⁴. However, all scale factors which are obtained at forward and medium angles (except

⁴ Note that the 25.80 MeV data set had to be freely floated, when the self-consistency of the DENZ04 DB₊ was investigated using the K -matrix expansions, e.g., see Section 3.1, as well as Tables 1 and 2; therefore, the normalisation of this data set is questionable even when the data set is compared only to the rest of the DENZ04 π^+p measurements. This implies that the seemingly-poor reproduction of the absolute normalisation of this data set on the basis of the ZUAS12 solution is not an issue. Indeed, if the scale factor of 0.8150 of Table 2 is applied to this data set, its resulting absolute normalisation will be compatible with our ZUAS12 solution.

at 37.10 MeV) show that the experimental data systematically exceed the ‘theoretical’ values obtained on the basis of the optimal parameters of Ref. [3]. The discrepancies reach the 35% level, with an average around 15%.

On the other hand, the scale factors for free floating \hat{z}_j obtained in the case of the truncated DENZ04 DB₋ (Fig. 2) cluster nicely around the expectation value of 1. Therefore, the absolute normalisation of the DENZ04 DB₋ appears to be compatible with the results obtained in Ref. [3].

To summarise, the absolute normalisation of the DENZ04 DB₋ appears to be in good agreement with our ZUAS12 solution, as is the normalisation of the π^+p backward-angle data sets. Large effects in the normalisation of the π^+p data sets have been seen at forward and medium scattering angles ⁵.

The hadronic part of the πN interaction is dominant in backward scattering. The electromagnetic (em) contributions become more and more important with decreasing scattering angle, finally culminating in the Coulomb peak which governs the very-forward scattering. The conclusion we drew from the analysis of the DENZ04 DB₋ is that the ETH model, along with the optimal values of the model parameters (as obtained in the ZUAS12 solution) and the known em contributions, accounts for the normalisation of the data successfully. The conclusion we drew from the analysis of the DENZ04 DB₊ is that the hadronic part of the interaction, as deduced on the basis of the ZUAS12 solution, is successful in reproducing the normalisation of the experimental data in the backward direction. Due to the fact that the problems lie with the scale factors obtained at forward and medium scattering-angle values, the truncated DENZ04 DB₊ seems to indicate modifications in the em part of the πN interaction. However, the em parts of the two elastic-scattering reactions are intimately connected; one cannot modify one and leave the other intact. (This comment also applies to the hadronic part of the amplitude when involving the ETH model in the fits. The two elastic-scattering reactions are related via the crossing symmetry which the model obeys.) Additionally, the Physics of the Coulomb peak has been established since a very long time.

In view of these results, it is now understood why the fit of the ETH model to the DENZ04 data drifts. As there are no adjustable parameters in the em part of the πN interaction, the adjustable hadronic part attempts to compensate for

⁵ It must be added that the absolute normalisation is not the only problem of the DENZ04 measurements. When using the ZUAS12 solution as reference, the shapes of 11 out of the 29 data sets (after all the outliers are removed) do not pass the test for $p_{min} \approx 1.24 \cdot 10^{-2}$. The disagreement in shape is very pronounced in the π^+p medium-angle 37.10 MeV, in the π^-p medium/backward-angle 25.80 MeV, and in the π^+p medium-angle 43.30(rot.) MeV data sets. Interestingly, *none* of the backward-angle measurements of the DENZ04 DB₊ shows any inconsistency in shape when compared with the ZUAS12 solution.

unexpected features in the data in a kinematical region in which the sensitivity of the DCS to the hadronic part of the interaction is expected to be low.

Given all these problems, we are currently unable to include the experimental data of Ref. [1] in our database. Additionally, we would like to remark that the use of these data in low-energy PSAs will surely lead to bias. We recommend the reprocessing of the raw experimental data, and we hope that the findings of the present study will be helpful in this task.

4 Discussion

This paper presents the results of an analysis of the DENZ04 [1] low-energy $\pi^\pm p$ differential cross sections. Given the size of this data (a total of 546 data points), the self-consistency of these measurements must be addressed prior to their inclusion into our database, which we have carefully ‘cleaned up’ and analysed in our last two phase-shift analyses (PSA) of Refs. [2,3].

The DENZ04 data were analysed as if they comprised the entire $\pi^\pm p$ elastic-scattering database at low energies by following the method set forth in Ref. [2]. The analysis of the DENZ04 $\pi^+ p$ measurements on the basis of standard low-energy expansions of the s - and p -wave K -matrix elements led to the identification of eight outliers in a total of 275 data points (see Table 2), whereas that of DENZ04 $\pi^- p$ elastic-scattering measurements to the removal of three out of a total of 271 data points (see Table 4).

We subsequently subjected the truncated DENZ04 combined $\pi^\pm p$ elastic-scattering databases into a common optimisation scheme, using the ETH model [4]. The ability of the model to account for the low-energy πN interaction has been demonstrated during the past two decades of research in this field. To our surprise, the optimisation failed to yield reasonable values for the model parameters. The phase-shift solution, extracted from the fit to the DENZ04 data, is very odd when compared to the values obtained in Refs. [3,5].

We next tried to trace the origin of these problems by investigating the reproduction of the DENZ04 data on the basis of the results of our recent PSA [3]. We found out that the absolute normalisation of the DENZ04 $\pi^- p$ elastic-scattering data is in good agreement with our ZUAS12 solution, as is the normalisation of the DENZ04 $\pi^+ p$ data sets at backward angles. On the other hand, large effects in the normalisation of the DENZ04 $\pi^+ p$ data sets have been seen at forward and medium scattering angles, i.e., in the region where the electromagnetic (em) effects are important. Therefore, the DENZ04 data seem to suggest modifications of the em part of the $\pi^+ p$ reaction, whereas the DENZ04 $\pi^- p$ elastic-scattering data are compatible with the em part as it

currently stands. Given the relation between the em amplitudes for the two reactions, the two aforementioned suggestions are mutually incompatible.

The failure of the model to account for the DENZ04 data has been understood on the basis of the interpretation of Figs. 1 and 2. Given that there are no adjustable parameters in the em part of the πN interaction, the adjustable hadronic part attempts to compensate for unexpected features in the data. Attempting to model large differences in a region where the sensitivity of the DCS to the hadronic part of the interaction is expected to be low, the model parameters drift away from their ‘established’ values, to unreasonable ones.

It is surprising that the majority of the problems, associated with the low-energy πN database, pertain to the π^+p reaction. The initial complacency when the BERTIN76 [6] values appeared soon gave way to scepticism, as the research groups, performing experiments at the meson factories around the world, reported serious discrepancies with the shape and the normalisation of the measurements of Ref. [6]. The 1990s saw the formation of two πN camps, one adhering to the results of the old PSAs (which, apart from the 67.10 MeV set, seem to reproduce the BERTIN76 data), another using only the results obtained at meson-factory facilities. Various analyses were carried out, dealing with (and, in general, proving) the self-consistency of the database as obtained from the ‘modern’ experiments. During the recent times, one was under the impression that, at last, the low-energy πN database *is* (after the rejection of a few obvious outliers) self-consistent. Unfortunately, the data we have dealt with in this study prove otherwise.

Acknowledgements

We acknowledge helpful discussions with G.J. Wagner on several issues relating to the acquisition and to the treatment of the experimental data of the CHAOS Collaboration. We also acknowledge an interesting exchange of ideas with I.I. Strakovsky. We would like to thank W.S. Woolcock for his comments and suggestions.

References

- [1] H. Denz et al., Phys. Lett. B 633 (2006) 209-13; the values of the elastic-scattering cross section have been taken from the site: <http://tobias-lib.uni-tuebingen.de/dbt/volltexte/2004/1323/>.
- [2] E. Matsinos, W.S. Woolcock, G.C. Oades, G. Rasche, A. Gashi, Nucl. Phys. A 778 (2006) 95-123.

- [3] E. Matsinos, G. Rasche, ‘Analysis of the low-energy $\pi^\pm p$ elastic-scattering data’, submitted to Nuclear Physics A; available in <http://arxiv.org>.
- [4] P.F.A. Goudsmit, H.J. Leisi, E. Matsinos, B.L. Birbrair, A.B. Gridnev, Nucl. Phys. A 575 (1994) 673-706; more references on the development of the ETH model may be found at http://people.web.psi.ch/matsinos/0_home.htm.
- [5] R.A. Arndt, W.J. Briscoe, I.I. Strakovsky, R.L. Workman, Phys. Rev. C 74 (2006) 045205; SAID PSA Tool, available at <http://gwdac.phys.gwu.edu>.
- [6] P.Y. Bertin et al., Nucl. Phys. B 106 (1976) 341-54.
- [7] E. Matsinos, Phys. Rev. C 56 (1997) 3014-25.

Table 1

The list of outliers in the DENZ04 π^+p database. The rows indicate steps in the outlier-identification/elimination process. The columns indicate: the χ^2 value, the number of degrees of freedom NDF in the fit, and the worst data point at that step; the worst data point was then removed and the fit to the remaining data was made. No data can be marked for removal at step 9. The worst data point is identified on the basis of the corresponding pion laboratory kinetic energy T (in MeV) and the centre-of-mass scattering angle θ . The presence of an angular interval at step 6 indicates that the corresponding data set (i.e., the backward-angle data set at 25.80 MeV) was freely floated in the subsequent fits.

Step	χ^2	NDF	Worst data point (T, θ)
1	486.9	268	25.80, 165.78°
2	474.0	267	37.10, 93.16°
3	463.2	266	19.90, 20.35°
4	450.7	265	37.10, 54.59°
5	441.3	264	19.90, 84.16°
6	431.1	263	25.80, 150.48 – 167.46°
7	421.6	262	19.90, 42.75°
8	411.9	261	37.10, 169.23°
9	401.2	260	

Table 2

The data sets comprising the truncated DENZ04 π^+p database, the pion laboratory kinetic energy T (in MeV), the number of degrees of freedom $(NDF)_j$ for each data set, the scale factor z_j which minimises χ_j^2 , the values of $(\chi_j^2)_{min}$, and the p-value of the fit for each data set. The numbers of this table correspond to the final fit to the data using the K -matrix expansions (see Section 3.1).

T	$(NDF)_j$	z_j	$(\chi_j^2)_{min}$	p-value	Comments
19.90	5	1.0591	9.3280	0.0967	20.35° removed
19.90	25	0.9378	40.8771	0.0236	42.75, 84.16° removed
25.80	5	1.0359	12.7392	0.0260	165.78° removed, freely floated
25.80	27	1.0739	33.0017	0.1970	
25.80	9	0.8150	16.3025	0.0608	
32.00	5	1.0299	6.3159	0.2767	
32.00	28	1.0595	44.4274	0.0252	
32.00	13	1.0149	17.9694	0.1587	54.59, 93.16° removed
37.10	8	0.8957	4.8626	0.7722	
37.10	26	0.8824	41.6963	0.0264	
37.10	12	0.8940	16.2488	0.1801	
43.30	12	0.9905	15.9922	0.1916	
43.30	28	0.9771	37.8442	0.1014	169.23° removed
43.30	13	0.9498	25.6257	0.0191	
43.30(rot.)	12	1.0214	23.2806	0.0254	
43.30(rot.)	27	0.9934	42.8706	0.0270	
43.30(rot.)	12	0.9512	11.8162	0.4606	

Table 3

The equivalent of Table 1 for the truncated DENZ04 π^-p elastic-scattering database.

Step	χ^2	NDF	Worst data point (T, θ)
1	388.2	264	25.80, 11.08°
2	371.5	263	43.30, 152.55°
3	358.9	262	25.80, 76.00°
4	350.2	261	

Table 4

The equivalent of Table 2 for the truncated DENZ04 π^-p elastic-scattering database.

T	$(NDF)_j$	z_j	$(\chi_j^2)_{min}$	p-value	Comments
19.90	6	1.0075	3.6609	0.7225	11.08° removed 76.00° removed 152.55° removed
19.90	25	0.9959	18.7750	0.8078	
25.80	6	1.0186	15.1307	0.0193	
25.80	37	1.0074	54.0761	0.0346	
32.00	5	0.9525	5.6430	0.3425	
32.00	40	0.9918	38.8705	0.5210	
37.10	9	0.9513	8.7386	0.4617	
37.10	41	0.9620	59.0529	0.0336	
43.30	12	1.0554	23.3763	0.0247	
43.30	38	1.0861	52.6021	0.0579	
43.30(rot.)	12	1.1054	21.0420	0.0498	
43.30(rot.)	37	1.1103	49.2107	0.0864	

Table 5

The reproduction of the DENZ04 data sets using the ZUAS12 solution as reference (i.e., yielding the ‘theoretical values’ y_{ij}^{th} in Eq. (1) of Ref. [3]). The columns correspond to: the pion laboratory kinetic energy T (in MeV), the number of degrees of freedom $(NDF)_j$ for each data set, the reported normalisation uncertainty [1], the scale factor z_j minimising χ_j^2 with its uncertainty Δz_j (combining in quadrature δz_j and the statistical uncertainty derived from Eq. (2) of Ref. [3]), the scale factor for free floating \hat{z}_j with its uncertainty $\Delta \hat{z}_j$, the minimal χ_j^2 value of the reproduction, the part of $(\chi_j^2)_{min}$ which corresponds to the statistical fluctuation in the data, and the part of $(\chi_j^2)_{min}$ which corresponds to the scaling of each data set as a whole. All definitions have been given in Section 2.2 of Ref. [3]. The table corresponds to the original DENZ04 data sets; all the data, which have been obtained at one condition (target configuration, energy), are assumed to comprise one data set in this table. The outliers, detailed in Tables 1 and 3, have not been removed.

T	$(NDF)_j$	δz_j	$z_j(\Delta z_j)$	$\hat{z}_j(\Delta \hat{z}_j)$	$(\chi_j^2)_{min}$	$(\chi_j^2)_{st}$	$(\chi_j^2)_{sc}$
π^+p scattering							
19.90	33	0.050	1.077(50)	1.078(50)	118.1	115.7	2.4
25.80	43	0.050	1.045(50)	1.046(50)	1208.8	1208.0	0.8
32.00	46	0.050	1.129(50)	1.130(50)	340.0	333.3	6.7
37.10	49	0.070	0.982(70)	0.982(70)	315.0	314.9	0.1
43.30(rot.)	53	0.070	1.061(70)	1.062(70)	235.6	234.8	0.8
43.30(rot.)	51	0.070	1.039(70)	1.039(70)	228.5	228.2	0.3
π^-p elastic scattering							
19.90	31	0.050	0.963(50)	0.963(50)	54.8	54.3	0.5
25.80	45	0.050	1.019(50)	1.019(50)	160.5	160.4	0.1
32.00	45	0.050	1.041(50)	1.042(50)	68.6	67.9	0.7
37.10	50	0.070	0.999(70)	0.999(70)	84.7	84.7	0.0
43.30(rot.)	51	0.070	1.052(70)	1.052(70)	83.1	82.5	0.6
43.30(rot.)	49	0.070	1.087(71)	1.088(71)	96.0	94.5	1.6

Table 6

The equivalent of Table 5 in the case that the original 12 data sets are split in a total of 29 segments (see Section 3). The outliers, detailed in Tables 1 and 3, have not been removed. The last column indicates the angular interval (f: forward, m: medium, b: backward, m/b: combined medium and backward angles).

T	$(NDF)_j$	δz_j	$z_j(\Delta z_j)$	$\hat{z}_j(\Delta \hat{z}_j)$	$(\chi_j^2)_{min}$	$(\chi_j^2)_{st}$	$(\chi_j^2)_{sc}$	θ
π^+p scattering								
19.90	6	0.050	1.084(51)	1.087(51)	24.1	21.1	2.9	f
19.90	27	0.050	1.070(51)	1.072(51)	95.1	93.1	2.0	m
25.80	5	0.050	1.063(51)	1.065(51)	21.2	19.6	1.6	f
25.80	27	0.050	1.208(51)	1.213(51)	62.8	45.0	17.8	m
25.80	11	0.050	0.824(51)	0.819(51)	42.0	29.2	12.7	b
32.00	5	0.050	1.157(54)	1.199(56)	19.0	6.5	12.5	f
32.00	28	0.050	1.193(50)	1.197(50)	73.7	58.5	15.2	m
32.00	13	0.050	1.023(51)	1.024(51)	19.4	19.1	0.2	b
37.10	8	0.070	1.076(72)	1.080(72)	9.5	8.2	1.2	f
37.10	28	0.070	1.008(70)	1.008(70)	110.1	110.1	0.0	m
37.10	13	0.070	0.912(70)	0.911(70)	27.3	25.7	1.6	b
43.30	12	0.070	1.225(77)	1.314(83)	28.8	14.4	14.4	f
43.30	28	0.070	1.130(71)	1.132(71)	45.0	41.5	3.5	m
43.30	13	0.070	0.983(71)	0.983(71)	23.4	23.3	0.1	b
43.30(rot.)	12	0.070	1.265(77)	1.360(82)	48.7	29.2	19.5	f
43.30(rot.)	27	0.070	1.063(70)	1.064(70)	87.3	86.4	0.8	m
43.30(rot.)	12	0.070	0.984(71)	0.984(71)	10.2	10.2	0.1	b
π^-p elastic scattering								
19.90	6	0.050	1.004(51)	1.005(51)	3.6	3.6	0.0	f
19.90	25	0.050	0.940(51)	0.939(51)	31.0	29.5	1.5	m
25.80	7	0.050	1.011(50)	1.011(50)	38.5	38.5	0.0	f
25.80	38	0.050	1.026(50)	1.027(50)	118.9	118.7	0.3	m/b
32.00	5	0.050	0.992(52)	0.991(52)	4.4	4.4	0.0	f

Table 6 continued

T	$(NDF)_j$	δz_j	$z_j(\Delta z_j)$	$\hat{z}_j(\Delta \hat{z}_j)$	$(\chi_j^2)_{min}$	$(\chi_j^2)_{st}$	$(\chi_j^2)_{sc}$	θ
32.00	40	0.050	1.050(50)	1.050(50)	49.0	48.0	1.0	m/b
37.10	9	0.070	0.986(71)	0.985(71)	7.8	7.8	0.0	f
37.10	41	0.070	1.002(70)	1.002(70)	75.4	75.4	0.0	m/b
43.30	12	0.070	1.052(71)	1.054(71)	23.4	22.8	0.6	f
43.30	39	0.070	1.051(71)	1.052(71)	60.2	59.7	0.5	m/b
43.30(rot.)	12	0.070	1.102(71)	1.106(71)	21.2	19.0	2.2	f
43.30(rot.)	37	0.070	1.066(71)	1.069(72)	73.1	72.2	0.9	m/b

Table 7

The equivalent of Table 5 in the case that the original 12 data sets are split in a total of 29 segments (see Section 3). The outliers, detailed in Tables 1 and 3, have been removed. The last column indicates the angular interval (f: forward, m: medium, b: backward, m/b: combined medium and backward angles).

T	$(NDF)_j$	δz_j	$z_j(\Delta z_j)$	$\hat{z}_j(\Delta \hat{z}_j)$	$(\chi_j^2)_{min}$	$(\chi_j^2)_{st}$	$(\chi_j^2)_{sc}$	θ
$\pi^+ p$ scattering								
19.90	5	0.050	1.094(51)	1.098(51)	14.8	11.1	3.7	f
19.90	25	0.050	1.070(51)	1.072(51)	66.0	64.0	2.0	m
25.80	5	0.050	1.063(51)	1.065(51)	21.2	19.6	1.6	f
25.80	27	0.050	1.208(51)	1.213(51)	62.8	45.0	17.8	m
25.80	9	0.050	0.830(51)	0.830(51)	17.3	17.3	0.0	b
32.00	5	0.050	1.157(54)	1.199(56)	19.0	6.5	12.5	f
32.00	28	0.050	1.193(50)	1.197(50)	73.7	58.5	15.2	m
32.00	13	0.050	1.023(51)	1.024(51)	19.4	19.1	0.2	b
37.10	8	0.070	1.076(72)	1.080(72)	9.5	8.2	1.2	f
37.10	26	0.070	1.009(70)	1.009(70)	101.6	101.6	0.0	m
37.10	12	0.070	0.907(70)	0.906(70)	17.3	15.5	1.8	b
43.30	12	0.070	1.225(77)	1.314(83)	28.8	14.4	14.4	f
43.30	28	0.070	1.130(71)	1.132(71)	45.0	41.5	3.5	m
43.30	13	0.070	0.983(71)	0.983(71)	23.4	23.3	0.1	b
43.30(rot.)	12	0.070	1.265(77)	1.360(82)	48.7	29.2	19.5	f
43.30(rot.)	27	0.070	1.063(70)	1.064(70)	87.3	86.4	0.8	m
43.30(rot.)	12	0.070	0.984(71)	0.984(71)	10.2	10.2	0.1	b
$\pi^- p$ elastic scattering								
19.90	6	0.050	1.004(51)	1.005(51)	3.6	3.6	0.0	f
19.90	25	0.050	0.940(51)	0.939(51)	31.0	29.5	1.5	m
25.80	6	0.050	1.029(51)	1.030(51)	19.1	18.7	0.3	f
25.80	37	0.050	1.023(50)	1.024(50)	109.9	109.7	0.2	m/b
32.00	5	0.050	0.992(52)	0.991(52)	4.4	4.4	0.0	f

Table 7 continued

T	$(NDF)_j$	δz_j	$z_j(\Delta z_j)$	$\hat{z}_j(\Delta \hat{z}_j)$	$(\chi_j^2)_{min}$	$(\chi_j^2)_{st}$	$(\chi_j^2)_{sc}$	θ
32.00	40	0.050	1.050(50)	1.050(50)	49.0	48.0	1.0	m/b
37.10	9	0.070	0.986(71)	0.985(71)	7.8	7.8	0.0	f
37.10	41	0.070	1.002(70)	1.002(70)	75.4	75.4	0.0	m/b
43.30	12	0.070	1.052(71)	1.054(71)	23.4	22.8	0.6	f
43.30	38	0.070	1.050(71)	1.051(71)	51.4	50.8	0.5	m/b
43.30(rot.)	12	0.070	1.102(71)	1.106(71)	21.2	19.0	2.2	f
43.30(rot.)	37	0.070	1.066(71)	1.069(72)	73.1	72.2	0.9	m/b

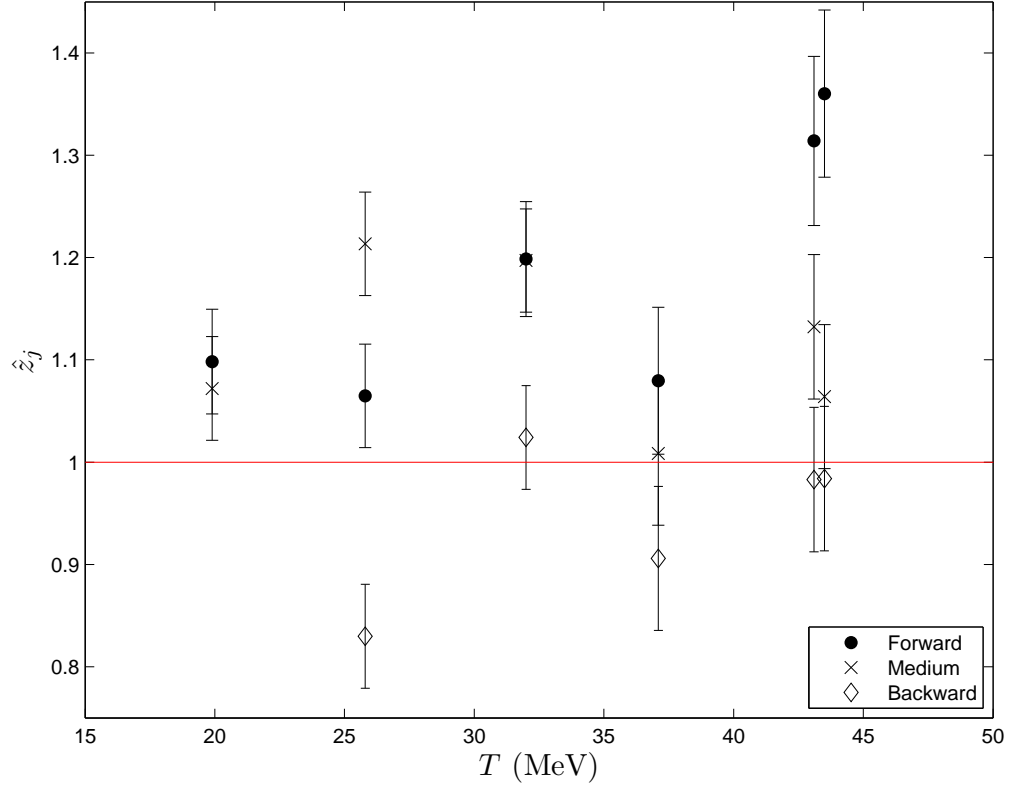


Fig. 1. The scale factors for free floating \hat{z}_j for the DENZ04 π^+p data, obtained on the basis of the ZUAS12 solution. To improve the display, the 43.30 MeV values are shown slightly shifted horizontally. The labels ‘forward’, ‘medium’, and ‘backward’ indicate the corresponding angular interval of the measurements.

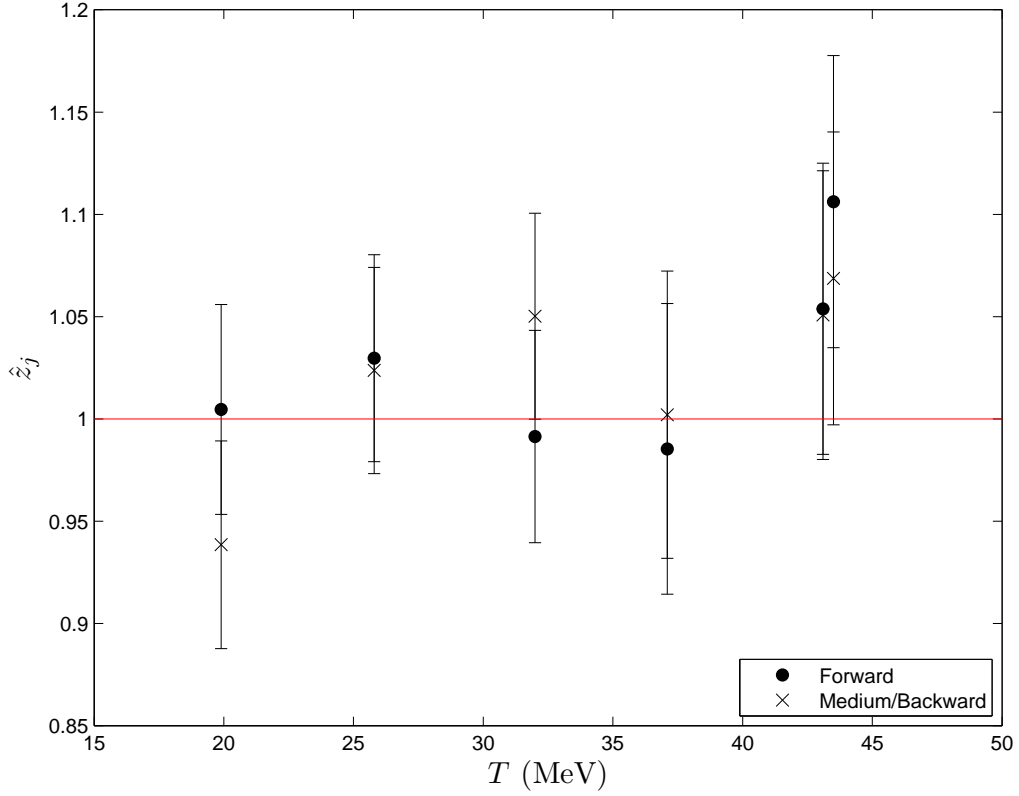


Fig. 2. The scale factors for free floating \hat{z}_j for the DENZ04 π^-p elastic-scattering data, obtained on the basis of the ZUAS12 solution. To improve the display, the 43.30 MeV values are shown slightly shifted horizontally. The labels ‘forward’ and ‘medium/backward’ indicate the corresponding angular interval of the measurements.